Free Vibration Analysis of Isotropic (SSSS) Non-Homogenous Triangular Plate with Varying Bi-Dimensional Thickness and Thermal Effect

A Synopsis

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Introduction

Vibration

A Mechanical oscillation of a body or particle from its position of equilibrium is called vibration. In this we have to study the motion of physical bodies to the forces acting on them. The fundamental theory in the mechanics of vibrations is space, time, and mass (or forces). When a body deviates from its original position, it tries to return to its initial position by the elastic property of the body's material. In current scenario, vibration analysis of complex-shaped structures is commonly faced in engineering and architectural fields. In aeronautical, marine, mechanical, and civil structural designs, regular, irregular, and complex shaped members are included to reduce costly material, lighten the loads, provide ventilation, and alter the resonant frequencies of the structures.

In the past vibration analysis of plates of various sizes and configurations has been extensively studied. The dynamic behavior of these structures is completely dependent on boundary conditions, geometrical shapes, various theories, different complicating effects and properties of used material.

Reasons of Vibration

The main reasons of vibration are as follows:

- Uneven distribution of forces in moving or rotating instruments
- External forces such as storm, tides, explosion, or earthquakes
- Friction between two bodies
- Magnetic or electric change
- Vehicular movement, etc

Requirements for Vibration

The main requirements of the vibration are as follows:

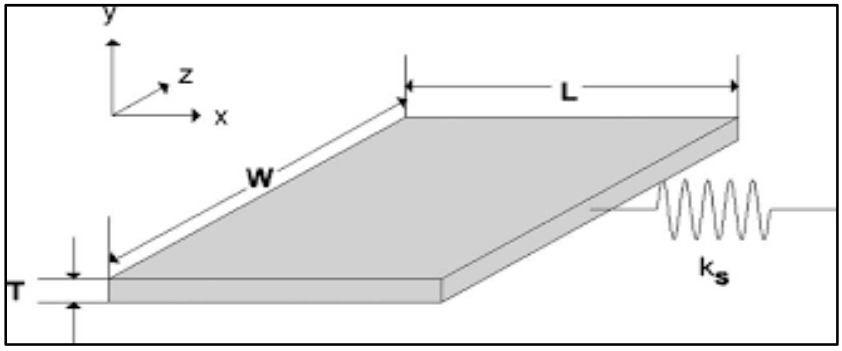
- There must be a restoring force.
- Average body position should be in balance.
- Inertia should be present (i.e. we should have mass).

Vibrations are of many types such as free vibrations, forced vibration, linear vibration, non-linear vibration, damped vibrations, un-damped vibrations etc. **Free vibration** is a vibration in which energy is neither added nor removed from the vibrating system. It vibrates forever with similar

amplitude. Whereas, **forced vibration** occurs when the energy is added to the vibrating system. **Linear** and **Non-linear** vibration depends on the behavior of the system. If mass, spring and dashpot behave in a linear manner on the system then the vibration caused in the form of linear vibration. On the other hand, if components of a vibratory system behave non-linearly then the vibrations produce in the form of non-linear vibrations. **Damped vibration** is one in which vibrating object loses its energy to the surroundings. The object loses energy due to resistance and as a result, the amplitude of vibration will eventually decay to zero. There are no free vibrations in nature; all are damped to some extent. While **un-damped vibration** suffers no energy loss. In present, we shall study free vibrations.

Plates

Plates are initially flat structural members bounded by two parallel plates, called **faces**, and surfaces, called an **edges** or boundary. The distance between the planes faces is called the **thickness** (h) -of the plate.



Generally plates are subjected to load conditions that cause deflections transverse to the plate. Geometrically they are bound either by straight or curved lines. Plates are of wide use in engineering industry. Many structures such as ships and containers require complete enclosure of plates without use of additional covering which consequently saves the material and labor. Nowadays, plates are generally used in architectural structures, bridges, hydraulic structures, pavements, containers, airplanes, missiles, ships, instruments and machine parts etc. The analysis of plates is mainly concern with the following parameters:

- Geometry of Plates (rectangular, square, circular, triangular, ellipse, cylindrical).
- Plate's Behaviour (isotropic, orthotropic, viscoelastic, homogeneous and non-homogeneous etc).
- Thickness Variation of Plate (linear, bilinear, circularly, exponentially, sinusoidal etc.)

- Applied load on plate (Static, dynamic, hydrostatic, thermal etc).
- Boundary Conditions (clamed, simply supported, free and fixed).

Visco-elastic Material

In this research, a plate behaves visco-elastically. The elastic and viscous behavior of material depends mainly on two parameters i.e. frequency and temperature. The properties of visco elastic materials such as rubber, concrete, soil, wood, cement etc. may exhibit linear visco-elasticity but metals and their alloys at high temperature exhibit non linear visco-elasticity according to their mechanical behavior. Therefore investigation of the vibration problems in visco-elastic theory is required in many structural applications of these materials due to thermal effect. The study of visco-elastic plate based on vibration is very much comprehensive due to various geometrical shapes with complication of anisotropy, non homogeneous, variable thickness, surrounding media, in plane force, large deflections elastic foundation, shear deformation, simple and mixed boundary conditions. Visco-elastic plate is of duralumin material, which is the alloy of aluminium. It contains 95% aluminium, 4% copper, 0.5% magnesium, 0.5% manganese.

Boundary Conditions

Generally, there are different types of boundaries considered for a plate and they are:

- Clamped boundary Conditions
- Simply Supported boundary Conditions
- Mixed boundary Conditions
- Free boundary Conditions

Clamped Boundary Conditions:

If a plate is clamped at the boundary, then the deflection and the slope of the middle surface must vanish at the boundary. On a clamped boundary parallel to the y- axis at x = a, the boundary conditions are

$$W|_{x=a} = 0$$

$$\left.\frac{\partial W}{\partial x}\right|_{x=a} = 0$$

The boundary conditions on the clamped boundary parallel to the x axis at y = b are

$$W|_{y=b} = 0$$

$$\left.\frac{\partial W}{\partial x}\right|_{y=b} = 0$$

Simply Supported Boundary Conditions:

A plate boundary that is prevented from deflecting but free to rotate about a line along the boundary, such as a hinge, is defined as a simply supported boundary. The conditions on a simply supported boundary parallel to the y axis at x = a are

$$W|_{x=a} = 0$$

$$M_x|_{x=a} = 0$$

Since the change of w with respect to the y coordinate vanishes along this boundary, the conditions become

$$W|_{x=a} = 0$$

$$\left.\frac{\partial^2 W}{\partial x^2}\right|_{x=a} = 0$$

On a simply supported boundary parallel to the x axis at y = b, the change of w with respect to the x coordinate vanishes, thus the conditions along this boundary are

$$W|_{y=b} = 0$$

$$M_y\big|_{y=b} = 0$$

Mixed Boundary Conditions:

Consider the plate to be simply supported on two opposite side and clamped on the other two sides at y = 0 and y = b. The boundary conditions for such a type of mixed boundary is

$$W|_{y=b} = 0$$

$$M_y\big|_{y=b} = 0$$

On the simply supported boundary parallel to the y axis the boundary condition at x = 0 and x =a

$W|_{x=a} = 0$

$$M_x|_{x=a} = 0$$

Free Boundary Conditions:

In the most general case, a twisting moment, a bending moment, and a transverse shear force act on a boundary of a plate. A boundary with all three of these stress resultants vanishing is defined as a free boundary. The boundary conditions on a free boundary parallel to the x axis at y = b is

$$M_y\big|_{y=b} = 0$$

$$M_{xy}\big|_{y=b} = 0$$

The boundary conditions on a free boundary parallel to the y axis at x = a

$$M_x|_{x=a} = 0$$

Literature Review

In the twentieth century, many researchers had studied the vibrational behavior of various types of plates.Much work has been published in the last five decades on the vibration of rectangular, Parallelogram and circular plates, but triangular plates are receiving comparatively less attention from researchers. The significance of investigation of decreased triangular plates is because of its application in assembling of boats, airplanes, submarines, and so on. This paper is a first endeavor to think about the vibration of triangular plates of changing thickness alongside the temperature variety.

In the changing situation of the advanced innovation, it is required to grow increasingly hypothetical and scientific models by which pointless and undesirable vibration of the machines or structures might be controlled. An assortment of research papers distributed in diaries, monographs, books, and so forth, in most recent couple of decades is exhibited as follows:

Leissa distributed a monograph dependent on the vibration of various kinds of the plates and gave a great deal of numerical data about the recurrence parameters for various states of plates with various limit conditions.

Gorman proposed another explanatory method for nothing vibration of essentially support right triangular plates to inspect Eigen values for the initial four methods of vibrations. The exhibited scientific methodology is relevant to right triangular plates with different kinds of limit conditions.

Mirza and Bijlani examined vibration of cantilever triangular plate with changing thickness by utilizing limited component technique. The creators assessed common frequencies furthermore; mode states of cantilevered triangular plates for different blends of four non-dimensional geometric variables, in particular the two headings feature to perspective ratio and two thickness ratio. The frequencies are arranged for different cases, which are higher, a couple of simple modes are displayed graphically.

Gorman studied the vibration of right triangular plates with all possible combinations of clamped boundary conditions and explicitly assisted. For the first four modes of free vibration of plates with a broad range of aspect ratio, the author used modified superposition method to investigate individual values and mode shape details.

Leissa and Jaber used the Ritz method to present the complete study of the frequencies as well as nodal patterns for completely free vibration of triangular plate. The functions of displacement were taken on in the form of algebraic polynomials. The nodal patterns and first six frequencies are introduced for 17 triangular plate configurations, possessed by varying the length ratios in a system.

With the help of the Rayleigh – Ritz method, **Liew and Chiam** discussed free vibration of isotropic and symmetrically laminated composite plates from the general triangular plate. The authors assessed geometry, material properties and lamination effects on the plate's natural frequencies.

For various boundary conditions, **Singh and Hassan** used the Rayleigh – Ritz method to obtain numerical solution to the vibration problem of triangular plates with uniform and arbitrarily varying thicknesses.

Sakiyama and Haung presented a free vibration analysis of right triangular plate with varying thicknesses and different boundary conditions. The writers found the triangular plate to be a kind of non-uniform-thick rectangular plate.

Cheung and Zhou studied free vibration of triangular plates with cantilevered and completely free isosceles based on an exact theory of three-dimensional elasticity. The Ritz method is used to derive the equation of one's own frequency from the strain energy and the plate's kinetic energy.

Chakarvarty provided a lot of information about the vibration properties of various plate types with different boundary conditions. The author provided authentic information with different boundary conditions for linear vibration of the elastic plates.

Zhang and Li suggested a solution approach for examining randomly formed triangular plates with elastically bound edges for vibration. The displacement function is expressed as a two-dimensional Fourier cosine series, complemented by various one-dimensional series to improve the displacement solution's convergence and accuracy.

Chaudhary and Falak have published a work for different materials on a laminated triangular plate with free-clamped boundary condition. The analysis was performed for the isotropic right triangular plate and for the triangular plate symmetrically laminated / composite.

Pradhan and Chakraverty addressed the effects of various physical and geometric parameters on the natural frequencies of the triangular equilateral plate subject to classical boundary

conditions. The numerical modeling is conducted using the Rayleigh–Ritz method to obtain the problem of own value in question.

Khanna and Kaur discussed first two modes of natural frequency parameters for the vibration of tapered rectangular viscoelastic isotropic plate under different thermal conditions in both tabular and graphic forms. The authors also addressed the effect of non-homogeneity with the Rayleigh–Ritz technique on free vibrations of a rectangular plate.

Justification

The justification for any research issue depends on its application across a variety of fields. In view of this, the researcher chose the subject entitled "**Free Vibration Analysis of Isotropic (SSSS) Non-Homogenous Triangular Plate with Varying Bi-Dimensional Thickness and Thermal Effect."** From the literature review it is observed that most work has been done for square, rectangular, circular and parallelogram plate. Much work has currently been done to justify vibration applications in engineering such as power plant equipment design, machinery, foundations, structures etc. The instrument fixed with machinery can lose its accuracy because of excessive vibrations. In the present scenario, design considerations relating to modifications of the vibrational behavior of automatic devices and structures are important. In addition, we know that plaques of inconsistent thickness are commonly used in recent technologies such as:

- Mechanical/Civil Engineering
- Aeronautical industry
- Automobile Industry
- Nuclear Power Stations
- Marine Engineering

In the aeronautical industry, plates having thermal constant and inconsistent thickness are of enormous significance because of their usefulness in aero plane wings. Since engineers have become more aware regarding importance of an elastic behavior of material. Hence, due to temperature variation on plates, we can neglect the known non-homogeneity that occurred in the material. Mathematical formulae are made to make the problem more useful and important for practical use in the field of engineering. One of the most important outcome in this research concerns with engineer and designer who will be gained with this study in the planning, adjustment and control techniques of designing and they can learn the observation, examination, for mechanical, civil engineering structures and aeronautical system that are optimized at the effects of vibrations.

Objectives

The main objectives of the research are:

- To analyze bi-linearly varying temperature effect on vibration of non-homogeneous isotropic triangular plate with inconsistent thickness linearly in two dimensions.
- To analyze bi-linearly varying temperature effect on vibration of non-homogeneous isotropic triangular plate with inconsistent thickness parabolically in two dimensions.
- To analyze bi-parabolically varying temperature effect on vibration of non-homogeneous isotropic triangular plate with inconsistent thickness parabolically in two dimensions.
- To analyze bi-parabolically varying temperature effect on vibration of non-homogeneous isotropic triangular plate with inconsistent thickness linearly in two dimensions.
- To analyze bi-linearly varying temperature effect on vibration of non-homogeneous isotropic triangular plate with inconsistent thickness linearly in one direction and parabolically in other direction.

Thesis Outline

In the proposed thesis an attempt is made to study the vibration on the non-homogeneous triangular plate under the effect of temperature and non-uniform varying thickness. The tentative wise chapter plan for the thesis was proposed as follows:

Chapter 1:

Introduction and Preliminaries

Chapter 2:

To analyze bi-linearly varying temperature effect on vibration of non-homogeneous isotropic triangular plate with inconsistent thickness linearly in two dimensions.

Chapter 3:

To analyze bi-linearly varying temperature effect on vibration of non-homogeneous isotropic triangular plate with inconsistent thickness parabolically in two dimensions.

Chapter 4:

To analyze bi-parabolically varying temperature effect on vibration of non-homogeneous isotropic triangular plate with inconsistent thickness parabolically in two dimensions.

Chapter 5:

To analyze bi-parabolically varying temperature effect on vibration of non-homogeneous isotropic triangular plate with inconsistent thickness linearly in two dimensions.

Chapter 6:

To analyze bi-linearly varying temperature effect on vibration of non-homogeneous isotropic triangular plate with inconsistent thickness linearly in one direction and parabolically in other direction.

Chapter 7:

Conclusions and scope for further research.

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